Subproblem Approach for Thin Shell Dual Finite Element Formulations

Vuong Q. Dang[1], Patrick Dular[1,2], Ruth V. Sabariego[1], Laurent Krähenbühl[3] and Christophe Geuzaine[1]
[1]Dept. of Electrical Engineering and Computer Science, University of Liège, Belgium, quocvuong.dang@ulg.ac.be
[2]Fonds de la Recherche Scientifique - F.R.S.-.FNRS, Belgium
[3]Université de Lyon, Ampère (CNRS UMR5005), École Centrale de Lyon, F-69134 Écully Cedex, France

***Abstract*—A subproblem technique is applied on dual formulations to the solution of thin shell finite element models. Both the magnetic vector potential and magnetic field formulations are considered. The subproblem approach developed herein couples three problems: a simplified model with inductors alone, a thin region problem using approximate interface conditions, and a correction problem to improve the accuracy of the thin shell approximation, in particular near their edges and corners. Each problem is solved on its own independently defined geometry and finite element mesh.**

I. Introduction

The solution by means of subproblems provides clear advantages in repetitive analyses and can also help in improving the overall accuracy of the solution [1], [2]. In the case of thin shell (TS) problems the method allows to benefit from previous computations instead of starting a new complete finite element (FE) solution for any variation of geometrical or physical characteristics. Furthermore, It allows separate meshes for each subproblem, which increases computational efficiency.

In this paper, a problem ($p = 1$) involving massive or stranded inductors alone is first solved on a simplified mesh without thin regions. Its solution gives surface sources (SSs) for a TS problem ($p = 2$) through interface conditions (ICs), based on a 1-D approximation [3], [4]. The TS solution is then corrected in a problem ($p = 3$) via SSs and VSs, respectively suppressing the TS representation and adding the actual volume representation, to take the actual field distribution of the field near edges and corners into account, which are poorly presented by the TS approximation. The method is validated on a practical test problem using a classical *brute force* volume formulation.

II. Definition of the Subproblem Approach

A. Canonical magnetodynamic or static problem

A canonical magnetodynamic or static problem p, to be solved at step p of the subproblem approach, is defined in a domain Ω, with boundary $\partial\Omega_p = \Gamma_p = \Gamma_{h,p} \cup \Gamma_{b,p}$. Subscript p refers to the associated problem p. The equations, material relations and boundary conditions (BCs) of the subproblems ($p = 1, 2, 3$) are:

$$\operatorname{curl} \boldsymbol{h}_p = \boldsymbol{j}_p\,,\ \operatorname{div} \boldsymbol{b}_p = 0\,,\ \operatorname{curl} \boldsymbol{e}_p = -\partial_t \boldsymbol{b}_p\,, \tag{1}$$

$$\boldsymbol{h}_p = \mu_p^{-1} \boldsymbol{b}_p + \boldsymbol{h}_{s,p}\,,\ \boldsymbol{j}_p = \sigma_p \boldsymbol{e}_p + \boldsymbol{j}_{s,p}\,, \tag{2}$$

$$\boldsymbol{n} \times \boldsymbol{h}_p|_{\Gamma_{h,p}} = \boldsymbol{j}_{su,p}\,,\ \boldsymbol{n} \cdot \boldsymbol{b}_p|_{\Gamma_{b,p}} = \boldsymbol{b}_{su,p}\,, \tag{3}$$

$$\boldsymbol{n} \times \boldsymbol{e}_p|_{\Gamma_{e,p} \subset \Gamma_{b,p}} = \boldsymbol{k}_{su,p}\,, \tag{4}$$

where $\boldsymbol{h}_p$ is the magnetic field, $\boldsymbol{b}_p$ is the magnetic flux density, $\boldsymbol{e}_p$ is the electric field, $\boldsymbol{j}_{s,p}$ is the electric current density, μ_p is the magnetic permeability, σ_p is the electric conductivity and $\boldsymbol{n}$ is the unit normal exterior to Ω_p. In what follows the notation $[\cdot]_{\gamma_p} = |_{\gamma_p^+} - |_{\gamma_p^-}$ expresses the discontinuity of a quantity through any interface γ_p (with sides γ_p^+ and γ_p^-) in Ω_p, defining interface conditions (ICs).

The fields $\boldsymbol{h}_{s,p}$ and $\boldsymbol{j}_{s,p}$ in (2) are VSs in the subproblem approach which can be used for expressing changes of permeability or conductivity (via $\boldsymbol{h}_{s,p}$ and $\boldsymbol{j}_{s,p}$, respectively). The fields $\boldsymbol{j}_{su,p}$, $\boldsymbol{b}_{su,p}$ and $\boldsymbol{k}_{su,p}$ in (3) and (4) are SSs and obtained from previous problems.

B. Constraints between subproblems

The constraints for the problems ($p = 1, 2, 3$) are respectively SSs and VSs. SSs are defined via the BCs and ICs of impedance-type boundary conditions (IBC). The TS model [4] has to be written as a subproblem following the already calculated inductor source field from problem ($p = 1$). The $\boldsymbol{b}$-formulation uses a magnetic vector potential $\boldsymbol{a} = \boldsymbol{a}_c + \boldsymbol{a}_d$ (such that $\operatorname{curl} \boldsymbol{a} = \boldsymbol{b}$). A similar decomposition is done for the $\boldsymbol{h}$-formulation, with $\boldsymbol{h} = \boldsymbol{h}_c + \boldsymbol{h}_d$. Fields $\boldsymbol{a}_c$, $\boldsymbol{h}_c$ and $\boldsymbol{a}_d$, $\boldsymbol{h}_d$ are respectively continuous and discontinuous through the TS. The weak $\boldsymbol{b}$- and $\boldsymbol{h}$-formulations involve the SSs in surface integral terms ($p = 2$), respectively

$$\langle [\boldsymbol{n} \times \boldsymbol{h}_2]_{\gamma_2}, \boldsymbol{a}'_c + \boldsymbol{a}'_d \rangle_{\gamma_2}\,,\ \langle [\boldsymbol{n} \times \boldsymbol{e}_2]_{\gamma_2}, \boldsymbol{h}'_c + \boldsymbol{h}'_d \rangle_{\gamma_2} \tag{5a-b}$$

with $\boldsymbol{a}_d$ and $\boldsymbol{h}_d$ defined as equal to zero on the side γ_2^- of the shell and $\gamma = \gamma_1^\pm = \gamma_2^\pm$; $\boldsymbol{a}'_d$, $\boldsymbol{h}'_d$, $\boldsymbol{a}'_c$ and $\boldsymbol{h}'_c$ are test functions. To explicitly express the field discontinuities, (5a-b) are rewritten:

$$\langle [\boldsymbol{n} \times \boldsymbol{h}_2]_{\gamma_2}, \boldsymbol{a}'_c \rangle_{\gamma_2} + \langle \boldsymbol{n} \times \boldsymbol{h}_2, \boldsymbol{a}'_d \rangle_{\gamma_2+} \tag{6}$$

$$\langle [\boldsymbol{n} \times \boldsymbol{e}_2]_{\gamma_2}, \boldsymbol{h}'_c \rangle_{\gamma_2} + \langle \boldsymbol{n} \times \boldsymbol{e}_2, \boldsymbol{h}'_d \rangle_{\gamma_2+} \tag{7}$$

The involved tangential fields in (6) and (7) are given by the TS model ($p = 2$) but some have to be corrected. The discontinuities in the first terms do not need any correction because solution ($p = 1$) presents no such discontinuities, i.e. $[\boldsymbol{n} \times \boldsymbol{h}_1]_{\gamma_1} = 0$ and $[\boldsymbol{n} \times \boldsymbol{e}_1]_{\gamma_1} = 0$. The tangential fields in the second terms have to be corrected with the opposed of the tangential contribution from solution ($p = 1$), i.e. $-\boldsymbol{n} \times \boldsymbol{h}_1$ and $-\boldsymbol{n} \times \boldsymbol{e}_1$. The resulting surface integral terms are correctly expressed via the weak formulations of problem ($p = 1$), thus rather via volume integrals, i.e.

$$\langle \boldsymbol{n} \times \boldsymbol{h}_1, \boldsymbol{a}'_d \rangle_{\gamma_2^+} = -(\mu_1^{-1} \operatorname{curl} \boldsymbol{a}_1, \operatorname{curl} \boldsymbol{a}'_d)_{\Lambda_1^+} - (\sigma_1 \partial_t \boldsymbol{a}_1, \boldsymbol{a}'_d)_{\Lambda_1^+} \tag{8}$$

$$\langle \boldsymbol{n} \times \boldsymbol{e}_1, \boldsymbol{h}'_d \rangle_{\gamma_2^+} = -(\mu_1^{-1} \partial_t \boldsymbol{h}_s, \boldsymbol{h}'_d)_{\Lambda_1^+} - (\mu_1^{-1} \partial_t \boldsymbol{h}_1, \boldsymbol{h}'_d)_{\Lambda_1^+} \tag{9}$$

with the volume integrals limited to a single layer of FEs Λ_1^+ touching $\gamma_2^+ = \gamma_1^+$, because they involve only the traces $\boldsymbol{n} \times \boldsymbol{a}'_d|_{\gamma_2^+}$ and $\boldsymbol{n} \times \boldsymbol{h}'_d|_{\gamma_2^+}$. Indeed, changing from μ_2 and σ_2 in a given subregion for TS problem (p =2) to μ_3 and σ_3 for volume problem (p=3) leads to the associated VSs

$$\boldsymbol{h}_{s,3} = (\mu_3^{-1} - \mu_2^{-1}) \boldsymbol{b}_2\,, \quad \boldsymbol{j}_{s,3} = (\sigma_3 - \sigma_2) \boldsymbol{e}_2\,. \tag{10}$$

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Once obtained, the TS solution ($p = 2$) is then corrected by problem ($p = 3$) that overcomes the TS assumptions [4]. It has to suppress the TS representation via SSs opposed to TS ICs, and to add the volumic shell via VSs that account for volumic change of μ_p and σ_p in problem ($p = 3$) that characterized the ambient region ($\mu_2 = \mu_0$, $\mu_3 = \mu_{volume}$, $\sigma_2 = 0$ and $\sigma_3 = \sigma_{volume}$). This correction will be shown to be limited to the neighborhood of the shell, which allows to benefit from a reduction of the extension of the associated mesh.

III. Application Example

The test problem is a shielded induction heater. It comprises two inductors (stranded or massive), a plate ($\mu_{r,plate} = 100$, $\sigma_{plate} = 1$ MS/m) in the middle, and two screens ($\mu_{r,screen} = 1$, $\sigma_{screen} = 37.7$ MS/m) (Fig. 1). It is first considered via a stranded inductor model (Fig. 2, top left, $\boldsymbol{a}_1$), adding a TS FE model (Fig. 2, bottom left, $\boldsymbol{a}_2$) that does not include the inductor anymore. Finally, a correction problem replaces the TS FEs with actual volume FEs (Fig. 2, top right, $\boldsymbol{a}_3$). The complete solution is shown as well (Fig. 2, bottom right, $\boldsymbol{a}_1 + \boldsymbol{a}_2 + \boldsymbol{a}_3$). Errors on the magnetic flux with the TS model between classical solution (FEM) and ($p = 1 + 2$) for both $\boldsymbol{b}$- and $\boldsymbol{h}$-formulations are shown in (Fig. 3a); they can nearly reach 85% in the end regions of the plate. Accurate local corrections are checked to be close to the complete volume FE solution (Fig. 3b). Significant TS errors are achieved through the relative correction of the eddy current as well (Fig. 4), up to 50% and 60% near the screen ends for ($\delta_{skindepth} = 0.919$ mm, $\mu_{r,plate} = 100$, f = 3kHz) and ($\delta_{skindepth} = 0.65$ mm, $\mu_{r,plate} = 200$, f = 3kHz) respectively. The proposed technique for TS FE and correction have been presented via a subproblem approach. It leads to accurate eddy current and magnetic flux distributions at the edges and corners of thin regions. All the steps of the method will be detailed, illustrated and validated in extended paper for both b- and h- formulations in 2D and 3D cases.

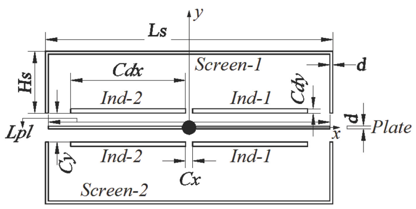

Fig. 1. Shielded induction heater ($d = 2 \div 6$ mm, $L_{pl} = 2$ m, $L_s = 2$ m+2d , $H_s = 400$ mm, $C_{dz} = 800$ mm, $C_{dy} = 10$ mm, $C_y = 200$ mm, $C_x = 50$ mm)

References

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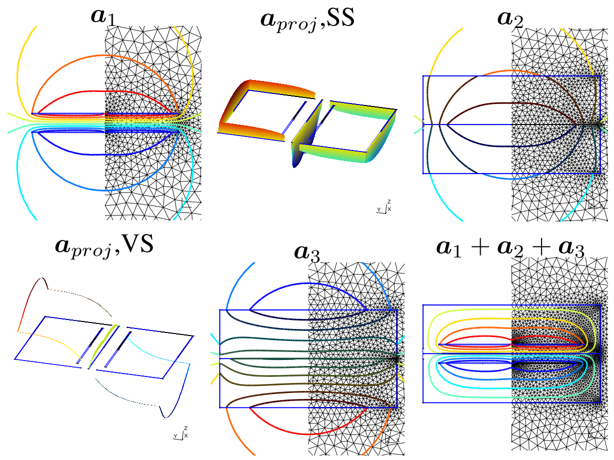

Fig. 2. Flux lines (real part) for the stranded inductor model ($\boldsymbol{a}_1$), thin shell added ($\boldsymbol{a}_2$), correction solution ($\boldsymbol{a}_3$) and the total solution ($\boldsymbol{a}_1+\boldsymbol{a}_2+\boldsymbol{a}_3$) with the different meshes used ($f = 1$ kHz, $\mu_{r,plate} = 100$, $\sigma_{plate} = 1$ MS/m). Projection of inductor solution ($\boldsymbol{a}_{proj}$,SS) in the TS, and of TS solution ($\boldsymbol{a}_{proj}$,VS) in the volumic shell.

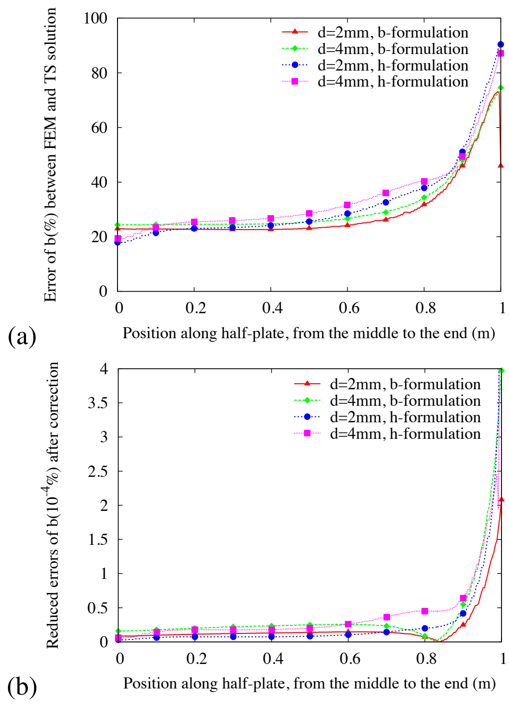

Fig. 3. Errors on the magnetic flux before correction (a) and after correction (b) along the plate for different thicknesses ($\mu_{r,plate} = 100$, $\sigma_{plate} = 1$ MS/m and f =1kHz).

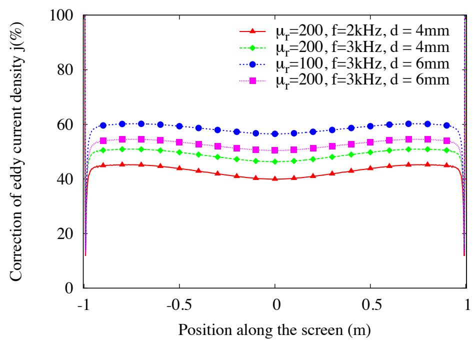

Fig. 4. Relative correction of the eddy current along the screen for effects of μ_r and frequency f ($\sigma_{plate} = 1$ MS/m).